CS 599 P1: Introduction to Quantum Computation | Boston University
Instructor: Alexander Poremba | Fall 2025

PRACTICE WORKSHEET #1

This is a practice worksheet—it will not be graded and is meant to refresh your memory of *complex numbers* and *linear algebra*. I strongly encourage you to work through these problems by yourself, ideally by **Tuesday, September 9th**—before the first homework assignment is out. **Solutions will be posted 09/07**.

Problem 1 (Complex numbers). A complex number $z \in \mathbb{C}$ is of the form $z = a + bi$ where $a, b \in \mathbb{R}$ and i is the *imaginary* unit with $i^2 = -1$. Here, $a = \mathrm{Re}(z)$ denotes the *real part* of z, and $b = \mathrm{Im}(z)$ denotes the *imaginary part* of z. Complex numbers have the following key properties:

- **Addition:** $(a + bi) + (c + di) = (a + c) + (b + d)i$
- **Multiplication:** $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$
- **Complex conjugate:** $\overline{z} = a - bi$
- **Modulus:** $|z| = \sqrt{a^2 + b^2}$
- **Unit circle:** Complex numbers with $|z| = 1$ lie on the unit circle (see Figure 1).
- **Rotations:** Multiplication by i rotates a point by $90°$ counterclockwise on the complex plane, whereas multiplication by -1 a point reflects across the origin.

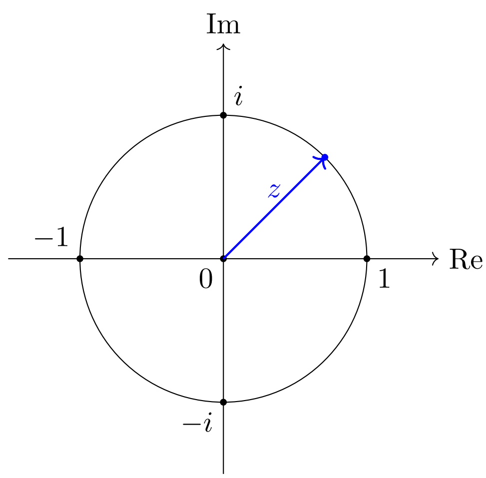

Figure 1: Any complex number $z \in \mathbb{C}$ can be written as $z = a + bi$, and can therefore be represented as a point on the complex plane. Here, the horizontal axis represents the real part, and the vertical axis represents the imaginary part. Whenever z has modulus $|z| = 1$, it lies on the unit circle, as pictured above.

Exercises:

1. Calculate the product $(1+i)(1-i)$.
2. Determine the complex conjugate and modulus of $z = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$.
3. Describe the geometric effect of multiplying $z = 1+i$ by $-i$.

Problem 2 (Complex vector spaces). A vector space over the complex numbers (or a *complex vector space*) is a set V of elements (called *vectors*) together with two natural operations: *addition* and (scalar) *multiplication*. The canonical example of a complex vector space is $V = \mathbb{C}^n$ with

$$\mathbb{C}^n = \left\{ \mathbf{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} : v_i \in \mathbb{C} \text{ for } i = 1, \ldots, n \right\}$$

where each of the n entries of a vector $\mathbf{v} \in \mathbb{C}^n$ is a complex number. Here, the addition operation over $\mathbb{C}^n$ is defined component-wise, such that

$$\mathbf{v} + \mathbf{u} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} + \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} = \begin{pmatrix} v_1 + u_1 \\ \vdots \\ v_n + u_n \end{pmatrix}, \quad \forall \mathbf{v}, \mathbf{u} \in \mathbb{C}^n.$$

Moreover, the multiplication operation by a scalar, $\alpha \in \mathbb{C}$, is defined as follows:

$$\alpha \cdot \mathbf{v} = \alpha \cdot \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} \alpha \cdot v_1 \\ \vdots \\ \alpha \cdot v_n \end{pmatrix}, \quad \forall \mathbf{v} \in \mathbb{C}^n.$$

Complex vector spaces, such as the canonical space $V = \mathbb{C}^n$, have a number of important characteristics and properties. We list some of these properties below:

- **Vectors:** a vector $\mathbf{v} \in \mathbb{C}^n$ typically represents a so-called *column vector*; however, there is also the notion of a *row* vector. Below, we compare these two kinds of representations:

$$\text{(column vectors)} \quad \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}; \qquad \text{(row vectors)} \quad \mathbf{v}^\mathsf{T} = (v_1, v_2, \ldots, v_n) = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}^\mathsf{T}.$$

- **Complex adjoint:** given a vector $\mathbf{v} \in \mathbb{C}^n$, we denote its complex adjoint by $\mathbf{v}^\dagger$. This is the vector we get by taking the complex conjugate of all of the entries of $\mathbf{v}$, and then applying the transpose of $\mathbf{v}$. In other words, we define $\mathbf{v}^\dagger := (\overline{\mathbf{v}})^\mathsf{T}$, which results in the *row* vector

$$\mathbf{v}^\dagger = \begin{pmatrix} \overline{v_1} \\ \overline{v_2} \\ \vdots \\ \overline{v_n} \end{pmatrix}^\mathsf{T} = (\overline{v_1}, \overline{v_2}, \ldots, \overline{v_n}).$$

- **Inner product:** In general, a complex vector space does not feature a multiplication rule between pairs of vectors $\mathbf{u}, \mathbf{v}$. The vector space $V = \mathbb{C}^n$, however, can be equipped with an *inner product*:

$$\langle \mathbf{v}, \mathbf{u} \rangle = \mathbf{v}^\dagger \cdot \mathbf{u} = \sum_{k=1}^{n} \overline{v_k} \cdot u_k, \quad \forall \mathbf{v}, \mathbf{u} \in \mathbb{C}^n.$$

  Because $\mathbf{v}^\dagger$ is a row vector and $\mathbf{u}$ is a column vector, this can be thought of as a form of matrix multiplication between a $(1 \times n)$ matrix and an $(n \times 1)$ matrix.

  The inner product $\langle \cdot, \cdot \rangle$ over $\mathbb{C}^n$ has the following properties:

  (Positivity)
  $\langle \mathbf{v}, \mathbf{v} \rangle > 0, \ \forall \mathbf{v} \in \mathbb{C}^n \setminus \{\mathbf{0}\}$

  (Skew symmetry)
  $\langle \mathbf{v}, \mathbf{u} \rangle = \overline{\langle \mathbf{u}, \mathbf{v} \rangle} \quad \forall \mathbf{v}, \mathbf{u} \in \mathbb{C}^n$

  (Linearity)
  $\langle \mathbf{v}, \alpha\mathbf{u} + \beta\mathbf{w} \rangle = \alpha\langle \mathbf{v}, \mathbf{u} \rangle + \beta\langle \mathbf{v}, \mathbf{w} \rangle$
  $\langle \alpha\mathbf{v} + \beta\mathbf{u}, \mathbf{w} \rangle = \overline{\alpha}\,\langle \mathbf{v}, \mathbf{w} \rangle + \overline{\beta}\,\langle \mathbf{u}, \mathbf{w} \rangle \quad \forall \mathbf{v}, \mathbf{u}, \mathbf{w} \in \mathbb{C}^n,\ \alpha, \beta \in \mathbb{C}.$

  Note that the conjugation rule in the linearity property only applies to the *first argument*.

- **Euclidean norm:** The Euclidean norm of a vector $\mathbf{v} \in \mathbb{C}^n$ is defined as $\|\mathbf{v}\| = \sqrt{\langle \mathbf{v}, \mathbf{v} \rangle}$.

- **Linear independence:** A set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_r\} \subseteq V$ is linearly independent if the only solution to the equation,

$$\lambda_1\mathbf{v}_1 + \lambda_2\mathbf{v}_2 + \cdots + \lambda_r\mathbf{v}_r = \mathbf{0},$$

  is $\lambda_1 = \lambda_2 = \cdots = \lambda_r = 0$, where $\lambda_i \in \mathbb{C}$.

- **Span:** The span of a set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_r\} \subseteq V$ is the set of all possible (complex) linear combinations of those vectors, i.e.,

$$\operatorname{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_r\} = \big\{\lambda_1\mathbf{v}_1 + \cdots + \lambda_r\mathbf{v}_r \ :\ \lambda_i \in \mathbb{C} \text{ for } i = 1, \ldots, r\big\}.$$

- **Basis:** A subset $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\} \subseteq V$ is called a *basis* of a complex vector space V, if the vectors $\{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ are all linearly independent and span the entire space V; in other words, every vector $\mathbf{v} \in V$ can be written as a linear combination

$$\mathbf{v} = \lambda_1\mathbf{b}_1 + \lambda_2\mathbf{b}_2 + \cdots + \lambda_n\mathbf{b}_n$$

  for some $\lambda_1, \ldots, \lambda_n \in \mathbb{C}$. If the vector space V admits a basis with n elements, we say V has dimension $\dim(V) = n$. An example of such a vector space is $V = \mathbb{C}^n$, which is n-dimensional and admits the basis $\mathcal{B} = \{\mathbf{e}_1, \ldots, \mathbf{e}_n\}$, where $\mathbf{e}_i$ are the canonical basis vectors

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \quad \cdots \quad, \quad \mathbf{e}_n = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}.$$

  Note that we will only consider finite-dimensional vector spaces in this class.

- **Orthogonal and orthonormal basis:** A set $\mathcal{B} = \{\mathbf{b}_1, \dots, \mathbf{b}_n\} \subseteq V$ is called an *orthogonal basis*, if any pair of basis vectors is orthogonal, i.e., $\langle \mathbf{b}_i, \mathbf{b}_j \rangle = 0$, for $i \neq j$. If additionally all vectors have unit norm $\|\mathbf{b}_i\|$=1, for all i, then $\mathcal{B}$ is called an *orthonormal basis*.

Exercise: Let $\mathbf{u} = \begin{pmatrix} 1 \\ i \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} i \\ 1 \end{pmatrix}$ be two-dimensional vectors in $\mathbb{C}^2$.

1. Compute their inner product $\langle \mathbf{u}, \mathbf{v} \rangle$.
2. Compute the norms $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$ of the two vectors.
3. Are $\mathbf{u}$ and $\mathbf{v}$ linearly independent?

Problem 3 (Linear operators). Recall that a complex-valued matrix $\mathbf{A} \in \mathbb{C}^{m \times n}$ is a *linear operator* $\mathbf{A} : \mathbb{C}^n \to \mathbb{C}^m$ that acts on the vector space $V = \mathbb{C}^n$; concretely, this means that, for any scalars $\alpha, \beta \in \mathbb{C}$, and any pair of vectors $\mathbf{u}, \mathbf{v} \in \mathbb{C}^n$, it has the property that

$$\mathbf{A} \cdot (\alpha \mathbf{u} + \beta \mathbf{v}) = \alpha \cdot \mathbf{A} \cdot \mathbf{u} + \beta \cdot \mathbf{A} \cdot \mathbf{v}.$$

Note that the same linearity property also extends to more general linear combinations of vectors in $\mathbb{C}^n$.

One important fact about linear operators is that they are *completely determined* by their action on a particular basis of the underlying vector space. We will now see why this is the case. Suppose that we fix a basis $\mathcal{B} = \{\mathbf{b}_1, \dots, \mathbf{b}_n\}$ of $\mathbb{C}^n$ (not necessarily the canonical basis). We want to show that $\mathbf{A} \in \mathbb{C}^{m \times n}$ is completely specified by how it transforms the set of basis elements:

$$\mathbf{b}_1 \mapsto \mathbf{A} \cdot \mathbf{b}_1, \quad \mathbf{b}_2 \mapsto \mathbf{A} \cdot \mathbf{b}_2, \quad \cdots, \quad \mathbf{b}_n \mapsto \mathbf{A} \cdot \mathbf{b}_n.$$

Because we fixed a particular basis $\mathcal{B}$, we can now write every vector $\mathbf{v} \in \mathbb{C}^n$ as a linear combination

$$\mathbf{v} = \lambda_1 \mathbf{b}_1 + \lambda_2 \mathbf{b}_2 + \cdots + \lambda_n \mathbf{b}_n$$

for some $\lambda_1, \dots, \lambda_n \in \mathbb{C}$. By using the linearity of $\mathbf{A} : \mathbb{C}^n \to \mathbb{C}^m$, we can expand this as follows:

$$\begin{aligned} \mathbf{A} \cdot \mathbf{v} &= \mathbf{A} \cdot (\lambda_1 \mathbf{b}_1 + \lambda_2 \mathbf{b}_2 + \cdots + \lambda_n \mathbf{b}_n) \\ &= \lambda_1 \cdot \underbrace{\mathbf{A} \cdot \mathbf{b}_1} + \lambda_2 \cdot \underbrace{\mathbf{A} \cdot \mathbf{b}_2} + \cdots + \lambda_n \cdot \underbrace{\mathbf{A} \cdot \mathbf{b}_n}. \end{aligned}$$

Thus, we can completely represent the action of any linear operator $\mathbf{A} : \mathbb{C}^n \to \mathbb{C}^m$ by recording the vectors

$$\mathbf{A}\mathbf{b}_1, \quad \mathbf{A}\mathbf{b}_2, \quad \cdots, \quad \mathbf{A}\mathbf{b}_n.$$

The *canonical basis* $\{\mathbf{e}_1, \dots, \mathbf{e}_n\}$ is a convenient choice, since we can very easily see that $\mathbf{A}\mathbf{e}_i = \mathbf{a}_i$ for

$$\mathbf{A} = \begin{pmatrix} | & | & & | \\ \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \\ | & | & & | \end{pmatrix}.$$

Once again, each column vector $\mathbf{a}_i$ precisely represents the action of $\mathbf{A} : \mathbb{C}^n \to \mathbb{C}^m$ on the i-th basis vector of the canonical basis $\{\mathbf{e}_1, \dots, \mathbf{e}_n\}$.

In class, we will often use the following properties of complex-valued matrices $\mathbf{A} \in \mathbb{C}^{m \times n}$:

- **Complex conjugate of a matrix:** For matrices, complex conjugation is applied element-wise:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mapsto \quad \overline{\mathbf{A}} = \begin{pmatrix} \bar{a}_{11} & \bar{a}_{12} & & \bar{a}_{1n} \\ \bar{a}_{21} & \bar{a}_{22} & \cdots & \bar{a}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \bar{a}_{m1} & \bar{a}_{m2} & \cdots & \bar{a}_{mn} \end{pmatrix}.$$

- **Transpose of a matrix:** The matrix transpose is given by the operation:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mapsto \quad \mathbf{A}^{\intercal} = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \vdots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}.$$

  Notice that the dimensions are now flipped: an $m \times n$ matrix becomes an $n \times m$ matrix.

- **Adjoint of a matrix:** For matrices, the complex adjoint is defined as $A^{\dagger} := (\overline{A})^{\intercal}$. In other words, it is the matrix we get by taking the complex conjugate, and then applying the transpose, i.e.,

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mapsto \quad \mathbf{A}^{\dagger} = \begin{pmatrix} \bar{a}_{11} & \bar{a}_{21} & \cdots & \bar{a}_{m1} \\ \bar{a}_{12} & \bar{a}_{22} & \cdots & \bar{a}_{m2} \\ \vdots & \vdots & \vdots & \vdots \\ \bar{a}_{1n} & \bar{a}_{2n} & \cdots & \bar{a}_{mn} \end{pmatrix}$$

Exercises: Suppose that that $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ is an arbitrary *orthonormal basis* of $\mathbb{C}^n$.

1. Show that the basis $\mathcal{B}$ has the property that $\langle \mathbf{b}_i, \mathbf{b}_j \rangle = \delta_{i,j}$, where $\delta_{i,j}$ is the Kronecker delta which is equal to 1, if $i = j$, and equal to 0, if $i \neq j$.

2. Show that any vector $\mathbf{v} \in \mathbb{C}^n$ can be written as

$$\mathbf{v} = \sum_{i=1}^{n} \langle \mathbf{b}_i, \mathbf{v} \rangle \cdot \mathbf{b}_i.$$

   Hint: To see this, expand the vector $\mathbf{v} \in \mathbb{C}^n$ as a linear combination of basis vectors in $\mathcal{B}$, i.e., let $\mathbf{v} = \lambda_1 \mathbf{b}_1 + \lambda_2 \mathbf{b}_2 + \cdots + \lambda_n \mathbf{b}_n$ for some $\lambda_1, \ldots, \lambda_n \in \mathbb{C}$. Plug in this expansion on both sides and use the properties of the inner product $\langle \cdot, \cdot \rangle$ to convince yourself that the two sides are equivalent.

3. Show that any matrix $\mathbf{A} \in \mathbb{C}^{n \times n}$ can be compactly written as

$$\mathbf{A} = \sum_{i,j=1}^{n} \langle \mathbf{b}_i, \mathbf{A}\mathbf{b}_j \rangle \cdot \mathbf{b}_i \cdot \mathbf{b}_j^{\dagger}.$$

   Hint: Try to check what happens when you hit the matrix $\mathbf{A}$ from the right with any vector $\mathbf{v} \in \mathbb{C}^n$ written as a linear combination $\mathbf{v} = \lambda_1 \mathbf{b}_1 + \lambda_2 \mathbf{b}_2 + \cdots + \lambda_n \mathbf{b}_n$ for some coefficients $\lambda_1, \ldots, \lambda_n \in \mathbb{C}$. Check that both sides of the equation give the same result.

Problem 4 (Eigenvalues and Eigenvectors). Let $\mathbf{A} \in \mathbb{C}^{n \times n}$ be a (square) matrix. A nonzero vector $\mathbf{v} \in \mathbb{C}^n$ is called an *eigenvector* of $\mathbf{A}$ if there exists a scalar $\lambda \in \mathbb{C}$ such that

$$\mathbf{A} \cdot \mathbf{v} = \lambda \cdot \mathbf{v}.$$

The scalar λ is called an *eigenvalue* of $\mathbf{A}$. Note that $\mathbf{A} \in \mathbb{C}^{n \times n}$ can have at most n distinct eigenvalues.

The set of eigenvalues can be obtained by solving the *characteristic* equation

$$\det(\mathbf{A} - \lambda \cdot \mathbf{I}_n) = 0,$$

where $\mathbf{I}_n$ is the $n \times n$ identity matrix.

We often encounter 2-dimensional complex matrices where the determinant has an especially simple form. Suppose that $\mathbf{M} \in \mathbb{C}^{2 \times 2}$ is a 2-dimensional matrix such that

$$\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Then, the determinant is given by the formula

$$\det(\mathbf{M}) = ad - bc.$$

Next, we explain how to find eigenvectors. For each eigenvalue λ, the corresponding eigenvectors are found by solving the corresponding equation

$$(\mathbf{A} - \lambda \cdot \mathbf{I}_n) \cdot \mathbf{v} = \mathbf{0}.$$

Remark 1. *Check page 7 for an example of how to calculate both eigenvalues and eigenvectors.*

The following set of exercises will help you practice how to calculate and find both eigenvalues and eigenvectors for two 2-dimensional matrices.

Exercises: Let $\mathbf{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \in \mathbb{C}^{2 \times 2}$ and $\mathbf{H} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \in \mathbb{C}^{2 \times 2}$.

1. Find the eigenvalues and eigenvectors of $\mathbf{X}$.
2. Find the eigenvalues and eigenvectors of $\mathbf{H}$.
3. Does $\mathbf{H}$ *commute* with $\mathbf{X}$, i.e., $\mathbf{XH} = \mathbf{HX}$?
4. Let $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{n \times n}$ be an arbitrary pair of anti-commuting matrices such that $\mathbf{AB} = -\mathbf{BA}$.
   Show that if they share a common eigenvector, then one of the respective eigenvalues must be 0.
   Hint: Check what happens when you hit both $\mathbf{AB}$ *and* $\mathbf{BA}$ *with the common eigenvector.*

Example: Eigenvalues and eigenvectors of a complex 2×2 matrix

Consider the 2-dimensional complex matrix

$$\mathbf{A} = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \in \mathbb{C}^{2\times 2}.$$

• **Step 1: Characteristic polynomial.** The eigenvalues are obtained by solving

$$\det(\mathbf{A} - \lambda \cdot \mathbf{I}_2) = 0.$$

Here, we first need to compute

$$\mathbf{A} - \lambda \cdot \mathbf{I}_2 = \begin{pmatrix} 1-\lambda & i \\ -i & 1-\lambda \end{pmatrix}.$$

Next, evaluating the determinant we can see that

$$\det(\mathbf{A} - \lambda \cdot \mathbf{I}_2) = (1-\lambda)(1-\lambda) - (i \cdot -i).$$

Since $i \cdot (-i) = -i^2 = 1$, we get

$$\det(\mathbf{A} - \lambda \cdot \mathbf{I}_2) = (1-\lambda)^2 - 1.$$

Expanding everything, we find

$$(1-\lambda)^2 - 1 = (\lambda^2 - 2\lambda + 1) - 1 = \lambda^2 - 2\lambda.$$

So the characteristic equation is precisely given by $\lambda^2 - 2\lambda = 0$.

• **Step 2: Solve for eigenvalues.**

$$\lambda(\lambda - 2) = 0 \quad \Rightarrow \quad \lambda_1 = 0, \ \ \lambda_2 = 2.$$

• **Step 3: Find eigenvectors.**

First, for $\lambda_1 = 0$, we need to solve:

$$\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

This gives $v_1 + iv_2 = 0$ and $-iv_1 + v_2 = 0$. Both reduce to $v_1 = -iv_2$. So an eigenvector is

$$\mathbf{v}^{(1)} = \begin{pmatrix} -i \\ 1 \end{pmatrix}.$$

Next, for $\lambda_2 = 2$, we need to solve

$$\begin{pmatrix} -1 & i \\ -i & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

This gives $-v_1 + iv_2 = 0$ and $-iv_1 - v_2 = 0$. Both reduce to $v_1 = iv_2$. So an eigenvector is

$$\mathbf{v}^{(2)} = \begin{pmatrix} i \\ 1 \end{pmatrix}.$$

Final Result: The matrix $\mathbf{A}$ has eigenvalues $\lambda_1 = 0$ and $\lambda_2 = 2$, and eigenvectors $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(2)}$, as above.